



Fostering Advanced Mathematical Thinking through Islamic Ethical Integration in Elementary Classrooms to Support Sustainable Development Goals (SDGs)

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ABSTRACT

This study investigates the potential of integrating Islamic ethical values into heuristic problem-based learning to foster Advanced Mathematical Thinking (AMT) skills among Islamic elementary students. Using an explanatory sequential mixed-methods design, the research involved 120 fifth-grade students from Madrasah Ibtidaiyah in a metropolitan city in Indonesia. The intervention employed the Heuristic Problem-Based Learning integrated with the Triplet Structure Model (HPbLTSM), emphasizing deep learning through inquiry, abstraction, representation, creativity, and proof. Results show that students with strong mathematical dispositions significantly improved their AMT skills, while even those with weaker dispositions benefited when ethical and collaborative frameworks were embedded. The study demonstrates that AMT is not exclusive to mathematically gifted students and that integrating Islamic values (such as ta'adub, musāwah, and taṭawwur wa ibtikār) enhances cognitive, ethical, and social growth. The findings align with the Sustainable Development Goals (SDGs), particularly SDG 4 (Quality Education) and SDG 16 (Peace, Justice, and Strong Institutions).

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1. INTRODUCTION

Many reports regarding mathematics education have been well-documented (Jose, 2022; Dermawan et al., 2022; Lagcao et al., 2023; Awofala & Olaniyi, 2023; Obafemi et al., 2024; Awofala & Agbolade, 2024). However, limited attention has been given to how higher-order thinking skills (particularly Advanced Mathematical Thinking (AMT)) can be cultivated from an early age within culturally and religiously contextualized settings. Advanced Mathematical Thinking (AMT) skills encompass five critical components of the mathematical learning process: representation, connection, communication, proof and reasoning, and problem-solving (Agustin et al., 2025a). In the context of Islamic elementary students in Madrasah Ibtidaiyah (MI), these competencies are aligned with the graduation competency criteria outlined by the Indonesian government through the Regulation of the Minister of Religion Number 2 of 2020 and the Regulation of the Ministry of Education and Culture Number 5 of 2022. To date, a clear and universally accepted definition of AMT remains absent. In this study, AMT skills are defined as an individual's cognitive ability to think numerically and logically, supported by psychological awareness and the capacity to progressively transform ideas from the concrete to the abstract, or from fundamental concepts to rigorous solutions. Developing AMT requires cultivating the habit of consistently transforming mathematical concepts into innovative and constructive logical outcomes.

AMT skills often manifest at least one of the three characteristics of epistemological obstacles described by Duroux. These obstacles arise from preexisting cognitive frameworks or knowledge structures that hinder the acquisition of new understanding. Overcoming such barriers requires not only high mathematical creativity but also a strong mathematical disposition, which encompasses convictions, perseverance, and self-efficacy that motivate students to engage deeply with mathematical challenges (Khalil & Prahmana, 2024). In this study, mathematical disposition is considered a key factor influencing students' thinking and the effective application of mathematics.

Islamic elementary students are nurtured within an educational environment that embodies the principles of the rahmatan lil alamin profile, which integrates ten values: ta'adub, qudwah, muwaṭānah, tawassuṭ, tawāzun, i'tidal, musāwah, syūra, tasāmuh, and taṭawwur wa ibtikār. Ta'adub, derived from adab, emphasizes cultivating civility before pursuing knowledge, as advised by Umar bin Khattab: "ta'addabū tsumma ta'allamū." Qudwah, based on QS Al-Ahzab [33]:21, refers to exemplary attitudes and morals to be emulated. Muwaṭānah reflects good citizenship and civic responsibility. Tawassuṭ, grounded in QS Al-Baqarah [2]:143, encourages moderation and adaptability in addressing religious and social matters, while tawāzun emphasizes balancing worldly and spiritual concerns, exemplified in the prayer "Rabbana atina fid-dunya ḥasanah wa fil-akhirati ḥasanah."

Other values are internalized through habitual practice. I'tidal signifies uprightness and consistency while maintaining openness to novelty. Musāwah, inspired by QS Al-Hujurat [49]:13, reflects equality in life and education. Collaborative learning is supported by syūra, as guided by QS Asy-Syūrā, emphasizing deliberation in decision-making. Social coexistence requires tasāmuh, or tolerance. Finally, taṭawwur wa ibtikār fosters innovation and openness, aligning students with civilizational progress. Integrating these values within mathematical learning provides a holistic foundation for cultivating AMT skills in Islamic elementary students.

Previous research underscores the importance of this study. Several researchers (Ardianto et al., 2024) found that students in Islamic Indonesian schools struggle to translate mathematical statements into corresponding concepts and procedures. Several researchers

(Agustin *et al.*, 2024a) reported that students often lack strong mental representation skills, including verbal, non-numeric symbolic, and visual representations. Additionally, Several researchers (Agustin *et al.*, 2024b) noted a lack of activities that engage students in mathematics while helping them understand the relationships between different ideas. Students also face difficulty viewing problems from multiple perspectives, which results in repetitive, mechanical solutions and challenges in formulating effective problem-solving strategies. Consequently, students struggle not only with discovering new connections between concepts but also with integrating established ideas to apply them in novel contexts. Several researchers (Stylianides & Stylianides, 2022) further highlighted that students encounter difficulties in developing mathematical proofs, particularly in the initial stages of the proof process and in connecting concepts to the desired conclusion.

There is ongoing debate regarding the feasibility of integrating AMT at the elementary school level (Selden & Selden, 2005). Some reports, the originator of AMT, support its introduction beginning in elementary education. Traditionally, AMT has been reserved for mathematically gifted students, such as Stephanie (Maher & Martino, 2000) and 15 gifted students among 231 elementary students in Tasikmalaya (Patmawati & Prabawanto, 2022). Recently, Several researchers (Agustin *et al.*, 2025a) demonstrated that AMT skills can be implemented in public elementary schools; however, their integration in MI remains unexplored. The contextual and abstract presentation of mathematical problems within the HPbLTSM learning model adds novelty to this study, allowing for a detailed analysis of AMT skills among Islamic elementary students. The findings highlight that HPbLTSM effectively transforms learning standards into actionable guidelines, facilitating the integration of best practices into classroom culture and challenging the assumption that AMT is exclusive to mathematically gifted students. This study adds information regarding current issues in sustainable development goals (SDGs).

2. METHODS

This study employed an explanatory sequential mixed-methods design, beginning with a quantitative phase followed by a qualitative phase. The objective was to generate statistical insights and subsequently contextualize them through comprehensive qualitative analysis. The quantitative phase used a quasi-experimental post-test-only control group design to mitigate pre-test bias, reduce the Hawthorne effect, focus on the treatment effect, and uphold internal validity. This design minimizes pretest sensitization and the reactive effects of the pretest.

A total of 120 fifth-grade students from a Madrasah Ibtidaiyah in a metropolitan city in Indonesia were selected through purposive sampling to ensure relevance, homogeneity, and to substantiate the applicability of AMT at the elementary level. Data collection instruments included a mathematical disposition questionnaire (Agustin *et al.*, 2025a) and a Three Worlds Journeys mathematics worksheet (Agustin *et al.*, 2025b), both validated via expert review and pilot testing. Validation involved one doctoral graduate and three lecturers with over 10 years of teaching experience and more than five years of training future elementary educators. A 4-point Likert scale was used to exclude neutral responses. Reliability analysis produced Cronbach's alpha values between 0.841 and 0.947, indicating strong internal consistency.

The experimental group received instruction using the HPbLTSM model (Figure 1), whereas the control group followed an expository learning model. Group homogeneity was confirmed ($p = 0.274$), and teachers were paired based on experience, qualifications, and pedagogical training to minimize variability. HPbLTSM encourages students to form mental connections

necessary for conceptual understanding by manipulating words, objects, or symbols. It also addresses the characteristics and needs of Islamic elementary students, bridging their zone of proximal development (ZPD) through dialogue and collaborative learning.

Heuristic strategies guide problem-solving without guaranteeing exact solutions (Kaitera & Harmoinen, 2022). Strategies included: drawing pictures or models, creating tables, identifying patterns, constructing diagrams, using colors, working backward, and employing the guess-check-revise method. Figure 1 illustrates the integration of heuristic instruction with the Triplet Structure Model (TSM) and Problem-Based Learning (PbL) syntax, forming the H-PbL-TSM framework. PbL consists of: (i) understanding the problem, (ii) planning an attack, (iii) carrying out the plan, and (iv) reviewing the solution (Szabo et al., 2020). TSM structures arithmetic story problems into main and supporting sentences, associating three quantities: operand, operant, and result quantity.

Pilot testing with 30 students validated item effectiveness, established appropriate difficulty levels (0.561–0.672), and demonstrated high discrimination indices (0.678–0.830). Statistical assumptions were satisfied: normality (Shapiro–Wilk, $p = 0.468$) and homogeneity (Levene’s, $p = 0.267$). The qualitative phase employed triangulated data sources, including worksheets, observation notes, instructional modules, and reflective journals, to ensure credibility and confirmability. All data were anonymized and securely archived for five years. These rigorous procedures collectively guarantee the validity, reliability, and trustworthiness of the study’s findings.

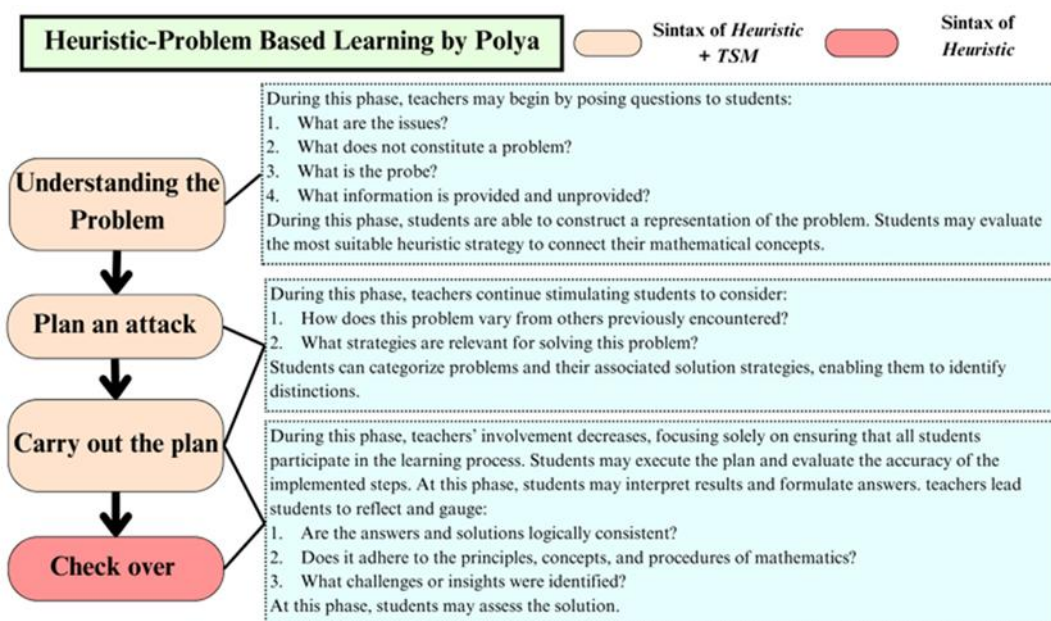


Figure 1. H-PbL-TSM Framework.

3. RESULTS AND DISCUSSION

3.1. AMT skills among Islamic elementary students

The results indicate that the interaction between learning models and students' mathematical disposition significantly affects AMT skills. Students with a strong mathematical disposition achieved the highest AMT scores (mean = 8.4) when taught using the HPbLTSM model, while those with weaker dispositions performed slightly better (mean = 8.3) under the expository model. Conversely, students with strong dispositions scored lower (mean = 4.5) in the expository model, and students with weak dispositions scored the lowest (mean = 6.0) in

the HPbLTSM model. A two-way ANOVA confirmed that these differences were statistically significant ($p < 0.001$).

Students with robust mathematical dispositions showed greater engagement with complex problem-solving tasks facilitated by the HPbLTSM model. This engagement enabled them to explore, represent, and connect mathematical ideas more effectively while developing reasoning and proof skills. **Table 1** summarizes the components of AMT skills and the corresponding assessment indicators, highlighting how HPbLTSM supports advanced thinking even among students with varying levels of mathematical disposition.

Table 1. AMT skills components and their indicators.

No	Components	Definition	Indicators
1.	Mathematical Representation	Communicating mathematical ideas (problems, processes, concepts, definitions, statements, mathematical connections, phenomena) in other forms.	Presenting mathematical ideas in other forms. Different ways of representing arguments include linguistic (e.g., verbal language), physical (e.g., concrete equipment), diagrammatic/pictorial, tabular (e.g., two-column format), and symbolic (e.g., algebra).
2.	Mathematical Abstraction	The construction process of abstract illustrations of the results of properties and concepts of mathematical objects.	Generalizing (identifying similarities in problems and information by bringing out specific aspects of a mathematical idea). Synthesizing (putting ideas together into a whole).
3.	Mathematical Creativity	The skill of finding new ideas by synthesizing originality, breaking down problems, and identifying patterns and solutions through relatively different perspectives to build unique and flexible mathematical ideas.	Fluency which refers to the generation of diverse ideas. Flexibility which involves producing new ideas in problem-solving. Elaboration or decomposition, which pertains to detailing the problem-solving process..
4.	Mathematical Proof	The systematic gathering and evaluation of arguments to verify a conjecture or hypothesis.	A set of statements that everyone agrees with, at least in the classroom. Valid arguments that everyone agrees with, at least in the classroom; Well-known versions of arguments that everyone agrees with, at least in the classroom.

The ability to transform mathematical concepts into various forms, linguistic, symbolic, diagrammatic, or physical, reflects proficiency in mathematical representation (Bruss, 2024; Kaitera & Harmoinen, 2022). This skill is closely linked to students' intrinsic motivation, self-confidence, and perseverance, qualities aligned with the Islamic values of tawassuṭ (moderation) and qudwah (exemplary behavior). Students often struggle to move beyond problem-solving methods provided by teachers (Woods & Copur-Gencturk, 2024). However, under the HPbL-TSM model, students are encouraged to explore, experiment with new strategies, and embrace change, allowing them to discover and even innovate more effective and efficient solutions. In this context, taṭawwur wa ibtikār reinforces students' curiosity and resilience in problem-solving.

Inquiry-based activities in HPbL-TSM also significantly foster mathematical abstraction. Students recognize patterns and integrate multiple concepts to develop overarching

principles, guided by curiosity and critical thinking. This abstraction process resonates with the Islamic principles of i'tidal (balanced judgment) and tawāzun (balance), as learners carefully evaluate information before concluding. Intuition and verification are essential in open-ended tasks, where students construct coherent conceptual frameworks (Bruss, 2024). Such activities cultivate independence and creativity, demonstrating how mathematical abstraction within AMT skills can be nurtured even with relatively flexible constraints.

Figure 2 depicts deep learning in mathematics: AMT skills among Islamic elementary students. The dynamic interplay between mathematical abstraction and representation forms the core of AMT skill development, symbolized as a yin-yang to emphasize their reciprocal and balanced relationship. Abstraction, driven by language-based illustration, involves the creation of generalized conceptual understandings derived from mathematical objects. Representation, on the other hand, highlights visual or physical depictions of ideas such as diagrams, symbols, and models (making abstract concepts more accessible and tangible). These two fundamental processes support mathematical creativity, allowing students to synthesize, elaborate, and explore ideas flexibly from multiple perspectives. As creativity matures, it culminates in mathematical proof, enabling learners to evaluate, justify, and articulate reasoning through logical argumentation. At the outermost layer, deep learning represents the culmination of these integrated processes, reflecting the full development of AMT. This progression from internal comprehension and representation to ideation and reasoning demonstrates that advanced mathematical thinking develops through organized, interconnected phases, progressively allowing students to engage with mathematics at deeper and more abstract levels. Active participation in proof construction is essential for achieving deep learning in mathematics (Bruss, 2024).

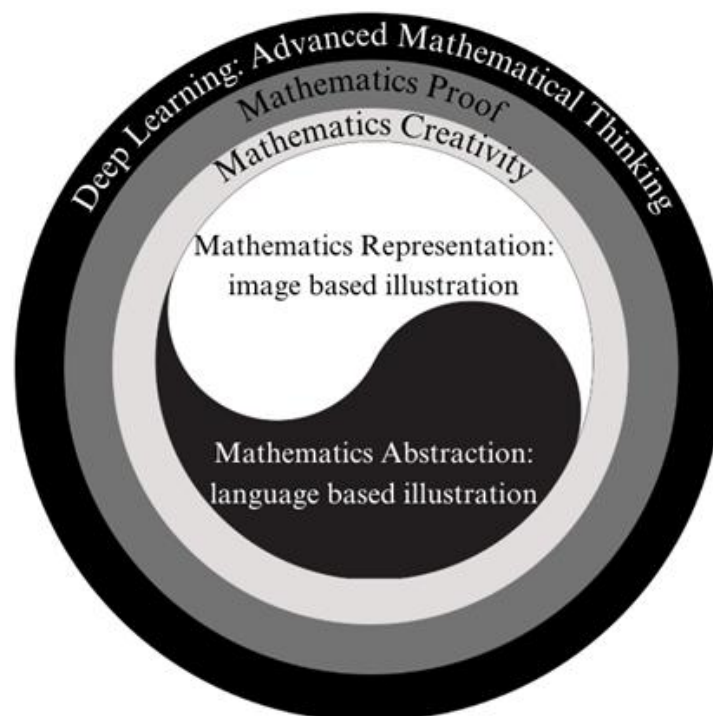


Figure 2. Deep Learning in Mathematics: AMT skills among Islamic elementary students.

Mathematical thinking demonstrates creativity through students' generation of multiple ideas (fluency), adaptation of strategies (flexibility), and elaboration of solutions (originality). HPbL-TSM supports this by presenting heuristic problem situations that require open-ended solutions. The integration of rahmatan lil alamin values, including musāwah (equality) and

syura (consultative collaboration), fosters inclusivity and encourages students to propose innovative solutions without fear of judgment, simultaneously activating and strengthening *taḥawwur wa ibtikār*. This approach promotes student autonomy in articulating ideas while proving their conjectures. The uniqueness of each student enriches the learning experience and facilitates progress through the ZPD via scaffolding. Collaborative learning enhances both engagement and mindful understanding.

Furthermore, the development of mathematical proof (the ability to substantiate conjectures and solutions) is strengthened by the collaborative and dialogic context of HPbL-TSM. Students systematically construct and evaluate arguments, reflecting the Islamic intellectual traditions of deliberation and evidence-based reasoning. Instructional scaffolding and group discussions, guided by principles such as *tabligh* (clarity in communication) and *amanah* (intellectual responsibility), further support student reasoning.

Conflicts in students' assumptions or conjectures frequently arise through engagement with mathematical phenomena, case analyses, and reasoning tasks. Within HPbL-TSM, proof tasks are integrated to support the ZPD through peer dialogue, independent work, and teacher guidance. Although much of the scaffolding is subtle and not directly observable, open-ended, non-routine problems stimulate critical engagement, occasionally causing temporary confusion that is later resolved through classroom debate (**Table 2**). For example, in lessons on multiplication and division of natural numbers, students A, B, C, and D engaged with a teacher (T) to navigate challenging tasks collaboratively. This process demonstrates how HPbL-TSM facilitates the natural scaffolding of AMT skills across all learning activities.

Table 2. Mathematical Proofs in an Islamic Elementary Mathematics Classroom.

Subject	Conversation	Points
A :	"Teacher, I've found numerous numbers that can replace the triangle."	(1)
T :	"Really?"	(2)
A :	"If I fill the triangle with the number 1, the result will be a fraction. If I fill the triangle with the number 45, I find the box of 960. However, I believe many other numbers can replace the triangle and box."	(3)
T :	"Can you find out how many numbers can replace the triangle?"	(4)
A :	"Many, teacher. Possibly infinite."	(5)
C :	"What if we replace the triangle with 960?"	(6)
D :	"Prioritizing parentheses means that if $9 \times 5 : 960$, the result will be a fraction."	(7)
B :	"Teacher, I have identified 2 numbers: 3,15. In conjunction with the 45 acquired by A, there exist 3 numbers that may substitute for the triangle."	(8)
E :	"Wait, we can change that triangle to 9, so we get the box as a result of 192. That means we already have 4 numbers. Do any of the other friends have any other numbers they can find?"	(9)
B :	"It is necessary to identify a factor that can exactly divide the product of 9 and 5. No additional numbers exist beyond the 4 identified."	(10)
A :	"It can be concluded that if the triangle contains the number 3, the box as an outcome is 64. If the triangle contains the number 9, the box is 192. When the triangle contains the value 15, the outcome is 320. Finally, if the triangle contains the number 45, the outcome is 960."	(11)

Students demonstrated the ability to express multiple mathematical statements observed by the class, validating their assertions indirectly. These statements were communicated through both verbal and written methods. Together, these three elements form the foundation of mathematical proof: (i) utilization of common knowledge that the community

already understands, including distinctions between numbers, basic rules of natural numbers, and multiplication with parentheses (illustrated in points 3 and 7); (ii) application of clear reasoning comprehensible to the community, showing that infinite number sentences identify factors under specific conditions (points 3, 5, and 10); and (iii) effective communication using spoken and written language that peers can understand (points 1, 3, 6, 7, 8, 9, 10, and 11). This process further stimulates representation, abstraction, and student creativity. Some reports describe three levels of conviction in verifying mathematical thinking: (i) convincing oneself, (ii) convincing a peer, and (iii) convincing an opponent, illustrating how AMT skills can be incorporated in elementary classrooms.

The value of *ta'adub* (civility) is observed in the courteous and thoughtful communication of Student A and peers with the teacher and classmates (points 1, 3, 5, 8), fostering an environment conducive to ethical inquiry (Harianto, 2022). *Syura* (collaborative discussion) and *musāwah* (equality) emerge prominently in group discourse, as students offer suggestions and attentively consider others' rationales (points 6, 8, 9), exemplifying inclusive participation. Leadership is demonstrated through Students A and B, who generalize patterns and guide group reasoning (points 3, 8, 11). *Tawassut*, *tawāzun*, and *i'tidal* (moderation, balance, justice) are evident in Students B and D's efforts to apply logical concepts to refine valid mathematical outcomes (points 7, 10), ensuring conclusions are equitable and precise. Simultaneously, *taṭawwur wa ibtikār* (innovation) manifests through varied strategies and rigorous examination of numerical substitutions (points 3, 5, 6, 9), promoting mathematical creativity. This interplay exemplifies advanced mathematical engagement and the internalization of Islamic values, showing that AMT growth through HPbL-TSM aligns with the moral-spiritual concept of *rahmatan lil 'alamin*.

In contrast, students with weaker mathematical dispositions performed better in traditional expository learning environments. Their engagement was more superficial, relying on structured guidance rather than independent inquiry. While their AMT indicators, including abstraction and creativity, were limited, they demonstrated proficiency in representation and procedural proof. These students heavily relied on discipline and role models, reflecting values such as *qudwah* and *i'tidal*, highlighting that Islamic values may manifest differently depending on context and individual learner profiles.

3.2. The context of HPbLTSM in Islamic elementary classrooms

Post-hoc analysis revealed that the experimental group outperformed the control group (mean = 3.86), whereas students with weaker mathematical dispositions showed a negative mean difference (-2.2634). These findings indicate that learning models must align with students' dispositions to effectively foster AMT skills. Traits such as curiosity, perseverance, and self-awareness, fundamental to the values of *rahmatan lil 'alamin* and closely linked to mathematical disposition, facilitate the development of advanced thinking from early education. HPbLTSM enhances these traits through collaborative inquiry and reasoning.

The HPbLTSM framework provides a structured approach to improve AMT skills in Islamic elementary education. Each stage of HPbLTSM carefully addresses AMT indicators, the cultural context of MI, and the competency levels outlined in Indonesia's deep learning curriculum. Differentiation in each phase is guided by a central focus: the initial phase prioritizes understanding the problem, followed by a shift to assessing students' competencies. The third phase emphasizes students' mathematical and psychological frameworks, while the fourth phase focuses on problem-solving strategies and evaluating solutions. Collaboration between teachers and students helps reduce learning gaps and promotes meaningful engagement.

HPbLTSM begins with fostering a deep understanding of problems, progresses to planning and executing strategies, and culminates in reflection on solutions, all while integrating religious and cultural principles. Mathematical disposition is evident in students' abilities to ask questions, generate answers, express creative ideas, and work collaboratively on open-ended problems (Ulya & Rahayu, 2021). The phased structure of HPbLTSM enhances mathematical comprehension by integrating inquiry, collaboration, hands-on activities, mathematical disposition, and Islamic character values.

In the initial phases, students build strong cognitive foundations and mathematical procedures, particularly when supported by favorable mathematical dispositions. The learning environment characterized by collaboration, reflection, and inquiry facilitates the internalization of *musāwah* (equality) and *taṭawwur wa ibtikār* (innovation and dynamism), guided by the principles of *syura* (consultation) and *tawassuṭ* (moderation) (Hadiyanto *et al.*, 2025). Students' intuitive abilities, allowing them to make independent choices, play a critical role in developing the cognitive mindset necessary for AMT skills, while low-complexity mental algorithms serve as initial scaffolds for cognitive reconstruction.

The comprehensive development provided by HPbLTSM enhances students' intuition, self-regulation, and cognitive restructuring, which are essential for meaningful engagement with AMT tasks. In advanced stages, students with strong mathematical dispositions demonstrate greater flexibility and originality, articulating their reasoning and proofs through effective communication. Inquiry-based activities and a solid mathematical disposition allow students to connect prior knowledge with newly learned concepts and relevant ideas. This process cultivates mathematical creativity, a key indicator of AMT skills.

Students with weaker dispositions often require scaffolding and emotional support to manage anxiety, particularly when confronted with non-routine or open-ended problems. Teacher facilitation in these contexts gradually adopts a less directive approach, emphasizing reflective guidance and conclusion drawing to enhance student autonomy and responsibility. Anxiety can affect cognitive functioning, disposition, and engagement, which can be mitigated through relaxation, cognitive reappraisal, and structured support strategies, helping students maintain *tawāzun* (balance) between thought, self, and emotion (Buckley *et al.*, 2016).

The third and fourth stages of HPbLTSM are crucial for fostering deeper mathematical creativity and proof skills. These phases emphasize problem presentation, hypothesis formulation, identification of knowledge gaps, and evaluation of procedural and conceptual understanding. Heuristic strategies on open-ended problems encourage deeper reasoning, enhance conceptual understanding, and strengthen executive functions, normative thinking, and the ability to challenge intuitive assumptions (Shtulman & Young, 2023; Woods & Copur-Gencturk, 2024).

Students progressively construct their own understanding through intuitive reasoning and deductive logic. Structured discussions and assigned tasks ensure inclusive participation and internal motivation. The collaborative nature of HPbLTSM supports the development of AMT across cognitive, affective, and psychomotor domains. Group engagement facilitates self-regulation, idea exchange, and bridging the gap between thought and action (Bruss, 2024). Students' interactions also foster key Islamic values, including *qudwah* (exemplary leadership), *tawāzun* (balance), and *ta'addub* (respect).

Empirical evidence indicates that the phased HPbLTSM structure enables students to confront misconceptions, adjust strategies, and develop abstract thinking. Students with strong dispositions show heightened self-awareness, improving mathematical proof and reasoning by reinforcing intellectual integrity. Appropriate questioning encourages

exploration of alternative solutions, while students with weaker dispositions benefit from teacher-guided tasks and emotional support through group work and scaffolded worksheets, facilitating representation skills.

Nevertheless, expository learning remains effective for students with weaker dispositions. Structured guidance and visual aids allow students to develop procedural knowledge and representation skills, although creativity and abstraction are less effectively cultivated. Tasks are uniform and repetitive, reducing anxiety but limiting independent exploration. Worksheets provide structured opportunities to convert real-world problems into tables, diagrams, or formulas, reinforcing understanding and reducing cognitive load.

Despite these benefits, expository methods have limitations in fostering independence, originality, and full AMT development. HPbLTSM, while most effective for students with strong dispositions, can support weaker learners through collaborative inquiry (syura), heuristic strategies, and emotional scaffolding. Group-based approaches enhance creativity by organizing heterogeneous students, encouraging questioning, contributions, and presentations (Catarino et al., 2019). **Figure 3** illustrates the operational flow of HPbLTSM, beginning with hands-on inquiry and culminating in deep learning and AMT skills.

Figure 3 illustrates the operational structure of HPbLTSM, integrating P5PRRA values into each phase of instruction. In the first phase, teachers stimulate students' thinking by presenting motives, problems, contexts, and objectives, while also introducing free play, learning rules, and representation activities. During this phase, values such as ta'adub (civility), qudwah (exemplary leadership), and muwatanah (civic responsibility) are cultivated as students are encouraged to question critically, listen attentively, and engage respectfully with the problem context. These activities ensure students are ethically anchored while examining known and unknown concepts, and they produce visual representations that embody the qualities, attributes, and ideas of mathematical entities.

In the second phase, emphasis shifts to tawassut (moderation), tawāzun (balance), and i'tidal (fairness), requiring students to evaluate methods, analyze examples, and avoid extremes in their reasoning. The third phase fosters musawah (equality), syura (collaborative consultation), and tasamuh (tolerance) through peer interaction, collective thinking, and openness to diverse approaches, promoting inclusive engagement and a community of inquiry. Finally, the fourth phase emphasizes taṭawwur wa ibtikār (continuous improvement and invention) as students refine and enhance their conclusions, consolidating mathematical proof while encouraging creativity, critical self-assessment, and accountability. This structure positions HPbLTSM as a comprehensive framework for cultivating advanced mathematical thinking while embedding the moral, civic, and spiritual principles of rahmatan lil 'alamin.

Visual representation plays a crucial role throughout the phases, even though explicit instruction in this skill is often limited. Engaging with objects and problems through images allows students to gain experiential understanding, while strategically guided representation strengthens their abstraction and problem-solving skills. Students' metacognitive development progresses from concrete to pictorial to abstract levels, supported not only by teacher and peer scaffolding but also by activities, assessments, and worksheets. During the first phase, students utilize concrete representations to express ideas, aligning with Bruner's concrete stage and Tall's embodied conceptual realm. In the second phase, they transition to pictorial or language-based representations, internalizing and encapsulating similarities, which reduces reliance on sensory perception and strengthens conceptual understanding. In the third phase, abstract representations are used to execute plans, test conjectures, and validate or refute statements using strategies such as trial-and-error and guess-check-revise, thereby enhancing abstraction, mathematical proof, and reasoning.

Overall, this phased, scaffolded approach systematically develops students' AMT skills while fostering inquiry, collaboration, creativity, and the internalization of Islamic ethical and civic values, creating a holistic environment where mathematical thinking and character formation proceed simultaneously.

Each of these levels explicitly cultivates and strengthens students' mathematical capacities for creativity. Throughout the process, conflicts among peers and moments of personal confusion naturally arise, prompting critical reflection and deeper engagement. At the core of this development lie representation and abstraction, encompassing both visual and verbal illustrations, which serve as foundational tools for mathematical reasoning. These processes seamlessly lead into creativity and proof, reinforced by dialogue, imagination, and logical coherence. Encapsulating all these components is deep learning, which signifies the culmination of integrated skills and represents the attainment of advanced mathematical understanding. This holistic progression demonstrates how HPbLTSM not only develops cognitive abilities but also nurtures collaboration, inquiry, and the ethical-spiritual values of *rahmatan lil 'alamin*, creating a rich environment for students to engage meaningfully with mathematics.

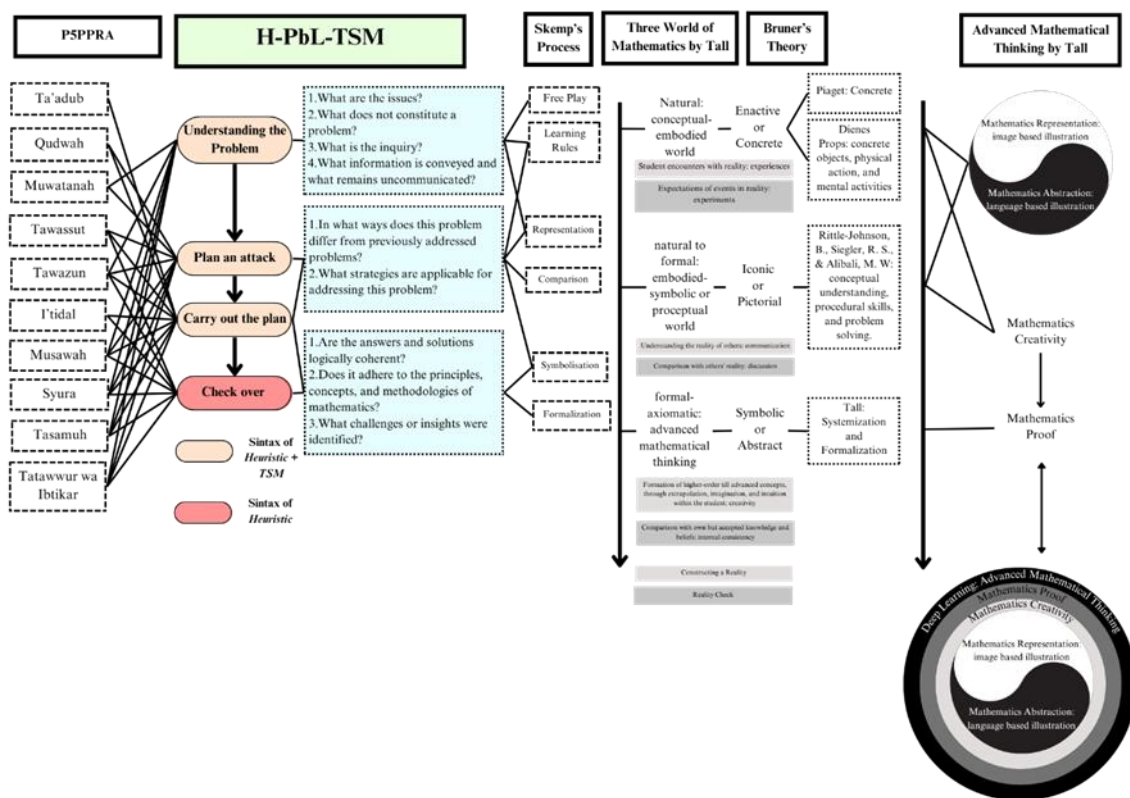


Figure 3. The Operational Flow of the H-PbLTSM Model in Islamic Elementary Classrooms.

3.3. *Rahmatan lil 'alamin* students' profile and Deep Learning Curriculum in Mathematics

PbL has long been recognized within educational communities, and the present study applies AMT in a MI setting that embodies the *rahmatan lil 'alamin* students' profile. By integrating collaborative (*syūra*) learning within PbL, the study aims to enhance students' mathematical proof and reasoning. Deep learning in mathematics classrooms unfolds primarily through the processes of proof and reasoning. Data from the study indicate that AMT development in MI mathematics is strengthened by fostering mathematical reasoning,

engaging students in proof tasks, and aligning with the standards of the mathematics learning process.

The connection skill emphasized in the mathematical learning process standard converges with mathematical abstraction in AMT. The overarching goal of AMT is to improve problem-solving skills within the framework of mathematics learning standards. Additionally, the Indonesian curriculum emphasizes eight characteristics of the graduate profile and ten values of the *rahmatan lil 'alamin* students' profile in MI. These elements overlap with the six competencies of 21st-century skills (6Cs), AMT skill indicators, and mathematics learning standards, creating an integrated educational structure. The ten foundational values of the *rahmatan lil 'alamin* profile serve as the basis for nurturing other character and cognitive dimensions. Within mathematics learning in MI, creativity and communication form the convergence point among these four elements (**Figure 4**).

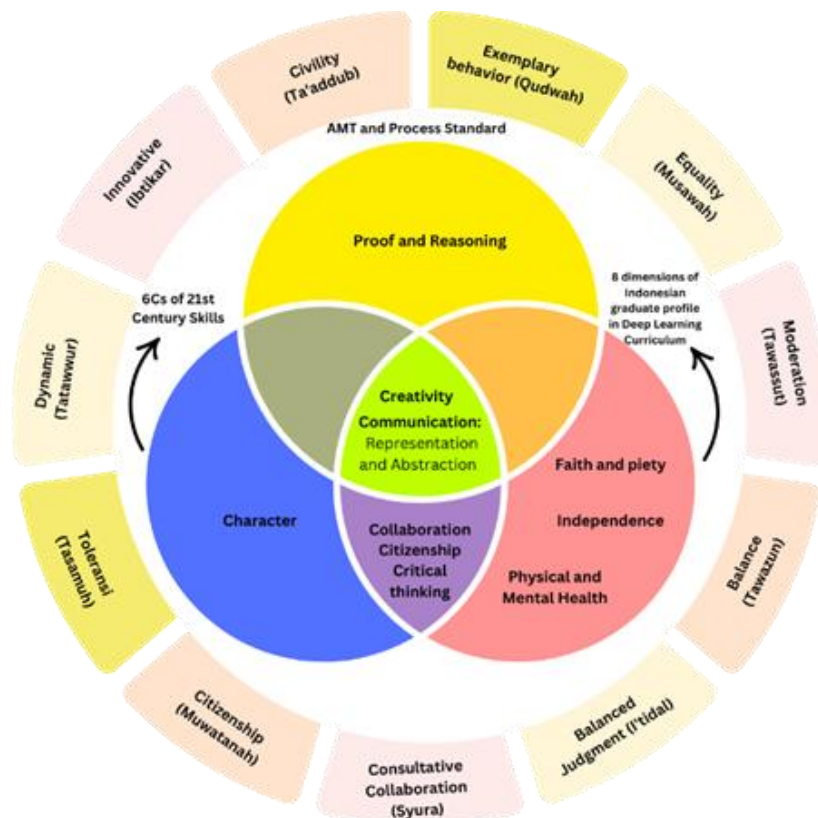


Figure 4. Relationship between AMT skills, mathematical learning process standard, 6Cs of 21st-century skills, 8 dimensions of Indonesian graduate profile in Deep Learning Curriculum, and *Rahmatan lil 'alamin* students' profile.

Figure 4 illustrates P5PRRA as a comprehensive framework merging Islamic values with AMT, 21st-century competencies, and the eight dimensions of the Indonesian graduate profile. Creativity and communication underpin mathematical representation and abstraction, forming the core of AMT while intersecting with character, critical thinking, collaboration, and citizenship. The framework consists of three clusters: (i) proof and reasoning, aligned with logic and justification, nurtured by ta'adub, qudwah, and muwatanah; (ii) character competencies, corresponding to the 6Cs and fostered through tasamuh, syura, and tatawwur; and (iii) socio-spiritual aspects of the graduate profile, anchored in tawassut, i'tidal, and tawāzun. The outer ring of values, including musawah, syura, and ibtikar, further supports inclusive, ethical, and innovative learning.

P5PRRA emphasizes values such as *ta'adub* (civility), *syura* (consultation), *ibtikar* (innovation), and *tawassut* (moderation). These values promote logical reasoning and abstraction essential for AMT while simultaneously cultivating collaboration, critical thinking, creativity, and character, thereby preparing students with the competencies needed in the 21st century. Socio-spiritual outcomes, including independence, faith, and mental well-being, are also reinforced, equipping students for both academic achievement and real-world ethical challenges. Embedding these principles throughout the HPbLTSM phases allows P5PRRA to create an integrated educational environment that nurtures intellectual, emotional, social, and spiritual growth, developing a generation that is both skilled and compassionate, globally responsible, and innovative.

The successful implementation of HPbLTSM is closely linked to teacher competency. Teachers serve as facilitators and role models and must possess a deep understanding of elementary mathematics concepts. Consequently, teacher learning communities, ongoing professional development, and collaborative research are essential to continually improve teaching strategies, particularly in question-based instruction and real-time classroom decision-making in Islamic elementary mathematics settings.

3.4. Contribution to the Sustainable Development Goals (SDGs)

The findings of this study provide significant contributions to several key SDGs, particularly SDG 4 (Quality Education) and SDG 16 (Peace, Justice, and Strong Institutions). Through the integration of the HPbLTSM model and Islamic ethical values in the learning process, this research promotes inclusive, equitable, and high-quality mathematics education. HPbLTSM encourages students to actively engage in higher-order thinking, fostering critical reasoning, creativity, and self-regulation, skills central to the SDG 4 target of ensuring effective learning outcomes.

Moreover, the alignment of AMT development with Islamic moral principles such as *musāwah* (equality), *syura* (consultation), and *ta'adub* (civility) supports peaceful and respectful classroom interactions, contributing to the values embedded in SDG 16. These ethical foundations promote a culture of tolerance, mutual respect, and collaborative inquiry, which are vital for nurturing socially responsible and morally grounded learners.

This integrative model also supports lifelong learning and character development, empowering students with the competencies needed to face complex global challenges. The study illustrates that advanced cognitive development and spiritual-ethical growth can coexist, highlighting the potential of culturally relevant pedagogies in advancing the global education agenda. By embedding ethical values into the learning process, the HPbLTSM approach offers a transformative educational model that supports not only academic excellence but also the formation of learners committed to justice, innovation, and sustainable development.

Finally, this study adds new ideas regarding SDGs, as reported elsewhere (**Table 3**). This study adds new information regarding Islam and Islamic education, as reported elsewhere (Kayode & Jibril, 2023; Darojah *et al.*, 2024; Daud, 2025; Jamiu, 2022; Al Husaeni & Al Husaeni, 2022; Hidayat *et al.*, 2024; Daud & Laguindab, 2025; Warren, 2025; Latifah *et al.*, 2025).

Table 3. Previous studies on SDGs.

No	Title	Reference
1	Efforts to improve sustainable development goals (SDGs) through education on diversification of food using infographic: Animal and vegetable protein	Awalussillmi et al., 2023
2	Experimental demonstration for teaching the concept of steam engine power plant to vocational students to support the Sustainable Development Goals (SDGs) and its comparison to Indonesian Merdeka curriculum	Fiandini et al., 2024
3	The influence of environmentally friendly packaging on consumer interest in implementing zero waste in the food industry to meet SDGs needs	Haq et al., 2024
4	Techno-economic analysis of solar panel production from recycled plastic waste as a sustainable energy source for supporting digital learning in schools based on SDGs and science-technology integration	Indra et al., 2025
5	AR technology to stimulate creativity in a zero-waste lifestyle in converting plastic waste to art products to support SDGs	Karyono et al., 2024
6	Analysis of student's awareness of sustainable diet in reducing carbon footprint to support SDGs 2030	Keisyafa et al., 2024
7	Enhancing professional readiness in vocational education through an integrative approach aligned with SDGs	Khamdamovna, 2025
8	Techno-economic analysis of eco-friendly bamboo-based paper production for child-friendly school media and SDGs	Kholik et al., 2025
9	SDGs in science education: Definition, literature review, and bibliometric analysis	Maryanti et al., 2022
10	Implementation of SDGs no. 12: Responsible production and consumption by optimizing lemon commodities	Maulana et al., 2023
11	Techno-economic feasibility of educational board game production from agro-industrial waste in support of SDGs	Mukmin et al., 2025
12	Computational calculation of adsorption isotherm characteristics of carbon microparticles prepared from mango seed waste to support SDGs	Nandiyanto et al., 2023
13	Silica microparticles with various sizes from bamboo leave waste for ammonia adsorption	Nandiyanto et al., 2024
14	Harnessing biomass for SDGs: Definition, bibliometric, application, opportunities, and challenges	Nandiyanto et al., 2025
15	Fabrication of resin-based brake pad from snake fruit peel as sustainable renewable resources to support SDGs	Nandiyanto et al., 2024
16	FTIR of pyrolysis of polypropylene microparticles and its chemical reaction mechanism... to support SDGs	Nandiyanto et al., 2024
17	Adsorption isotherm characteristics of calcium carbon microparticles prepared from chicken bone waste to support SDGs	Nandiyanto et al., 2023
18	Analysis of the application of Mediterranean diet patterns on sustainability to support SDGs	Nurnabila et al., 2023
19	Biomass composition (cassava starch and banana peels) on mechanical and biodegradability properties of bioplastics for supporting SDGs	Ragadhita et al., 2023
20	Biomass composition (cassava starch and banana peels) on mechanical and biodegradability properties of bioplastics for supporting SDGs	Ragadhita et al., 2022
21	Safe food treatment technology: The key to realizing the SDGs zero hunger and optimal health	Rahmah et al., 2024
22	Global warming: Promoting environmental awareness of senior secondary school students facing issues in the SDGs	Supriatna et al., 2024
23	Profile of sustainable science teacher at junior high schools in Riau towards the SDGs	Vilmala et al., 2022
24	School feeding program and SDGs in education: Linking food security to learning outcomes in Timor-Leste	Ximenes, 2025

4. CONCLUSION

This study confirms that Advanced Mathematical Thinking (AMT) is not an exclusive domain of mathematically gifted students but can be cultivated through contextually grounded instruction that integrates ethical, cultural, and cognitive dimensions. The application of the Heuristic Problem-Based Learning integrated with the Triplet Structure Model (HPbLTSM) significantly enhanced students' AMT skills (namely representation, abstraction, creativity, reasoning, and proof), particularly among those with strong mathematical dispositions. Moreover, the integration of Islamic ethical values through the P5PRRA framework promoted not only cognitive growth but also character development aligned with the rahmatan lil 'alamin profile. The findings demonstrate that embedding values such as ta'adub, syura, musāwah, and taṭawwur wa ibtikār in classroom practice fosters collaborative inquiry, reflective thinking, and responsible engagement. These outcomes support the achievement of Sustainable Development Goals (SDGs), especially SDG 4 (Quality Education) and SDG 16 (Peace, Justice, and Strong Institutions). Thus, HPbLTSM emerges as a transformative model for holistic mathematics education in Islamic elementary settings.

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6. AUTHORS' NOTE

The authors declare that there is no conflict of interest regarding the publication of this article. Authors confirmed that the paper was free of plagiarism.

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